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**Mathematics**  
**Higher level**  
**Paper 3 – sets, relations and groups**

Tuesday 10 November 2020 (afternoon)

1 hour

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**Instructions to candidates**

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

Consider two subsets,  $A$  and  $B$ , of a universal set  $U$ .

(a) Draw a Venn diagram to illustrate

(i)  $(A \cup B)'$ ;

(ii)  $A' \cup B'$ .

[2]

(b) Use the laws of set operations to show that  $A \cap (A \cap B)' = A \setminus B$ .

[3]

2. [Maximum mark: 10]

The relation  $R$  is defined on  $\mathbb{R} \times \mathbb{R}$  by  $(a, b)R(c, d)$ , if and only if,  $b - a = d - c$ .

(a) Prove that  $R$  is an equivalence relation.

[5]

(b) (i) Find two elements of  $\mathbb{R} \times \mathbb{R}$  in the same equivalence class as  $(1, 3)$ .

(ii) Describe geometrically the equivalence class containing  $(1, 3)$ .

(iii) Describe geometrically the equivalence classes of the relation  $R$ .

[5]

3. [Maximum mark: 5]

The function  $g : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \times \mathbb{R}$  is defined by  $g(x, y) = (xy, x + y)$ .

(a) Give an example to demonstrate that  $g$  is not injective.

[2]

(b) Show that  $g$  is not surjective.

[3]

4. [Maximum mark: 18]

The binary operation  $*$  is defined on the set  $S = \mathbb{R} \setminus \{1\}$  by  $a * b = a + b - ab$ , for all  $a, b \in S$ .

- (a) Find the identity element. [2]
- (b) Show that  $*$  is associative. [3]
- (c) Solve  $x * x * x * x = -8$ . [5]
- (d) Show that every element of  $S$  has an inverse. [2]
- (e) Hence show that  $\{S, *\}$  is a group. [3]

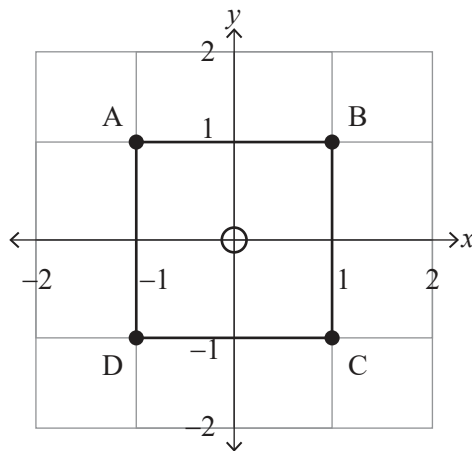
The set  $H = \{0, 2\}$ .  $\{H, *\}$  is a subgroup of  $\{S, *\}$ .

- (f) (i) Find  $4H$ , the left coset of  $H$  with respect to 4.
- (ii) Solve  $nH = 4H$ . [3]

Turn over

5. [Maximum mark: 12]

Consider the square ABCD with vertices at  $A(-1, 1)$ ,  $B(1, 1)$ ,  $C(1, -1)$  and  $D(-1, -1)$  as shown in the following diagram.



The set of vertices of the square is  $V = \{A, B, C, D\}$ . A reflection of the square in the line  $x = 0$  can be represented by the permutation  $(AB)(CD)$  which gives the final position of the vertices.

- (a) Describe the single geometric transformation represented by the permutation
  - (i)  $(ABCD)$ ;
  - (ii)  $(AC)$ . [4]
- (b) Find the permutation that represents
  - (i) a rotation of  $180^\circ$  about  $(0, 0)$ ;
  - (ii) a reflection in the line  $y = 0$ . [2]

The permutations representing the eight symmetries of the square form a group  $\{G, \circ\}$  where  $\circ$  is the operation of composition of permutations.

- (c) Find the single permutation that represents  $(ABCD) \circ (AC)$ . [2]

Let the identity permutation be represented by  $e$ .

- (d) Find a cyclic subgroup of  $G$  of order 4. [2]
- (e) Find a non-cyclic subgroup of  $G$  of order 4. [2]