Diploma Programme
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Mathematics
Higher level
Paper 3 – sets, relations and groups

Tuesday 10 November 2020 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

Consider two subsets, A and B, of a universal set U.

(a) Draw a Venn diagram to illustrate

(i) $(A \cup B)'$;

(ii) $A' \cup B'$. [2]

(b) Use the laws of set operations to show that $A \cap (A \cap B)' = A \setminus B$. [3]

2. [Maximum mark: 10]

The relation R is defined on $\mathbb{R} \times \mathbb{R}$ by $(a, b) R (c, d)$, if and only if, $b - a = d - c$.

(a) Prove that R is an equivalence relation. [5]

(b) (i) Find two elements of $\mathbb{R} \times \mathbb{R}$ in the same equivalence class as $(1, 3)$.

(ii) Describe geometrically the equivalence class containing $(1, 3)$.

(iii) Describe geometrically the equivalence classes of the relation R. [5]

3. [Maximum mark: 5]

The function $g : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ is defined by $g(x, y) = (xy, x + y)$.

(a) Give an example to demonstrate that g is not injective. [2]

(b) Show that g is not surjective. [3]

4. [Maximum mark: 18]

The binary operation $*$ is defined on the set $S = \mathbb{R} \setminus \{1\}$ by $a * b = a + b - ab$, for all $a, b \in S$.

(a) Find the identity element. [2]

(b) Show that $*$ is associative. [3]

(c) Solve $x * x * x * x = -8$. [5]

(d) Show that every element of S has an inverse. [2]

(e) Hence show that $\{S, *\}$ is a group. [3]

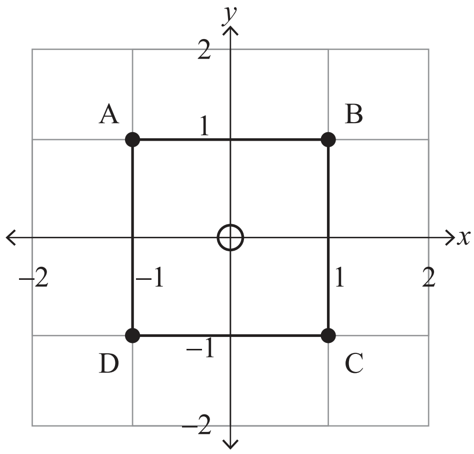
The set $H = \{0, 2\}$. $\{H, *\}$ is a subgroup of $\{S, *\}$.

(f) (i) Find $4H$, the left coset of H with respect to 4.

(ii) Solve $nH = 4H$. [3]

5. [Maximum mark: 12]

Consider the square ABCD with vertices at $A(-1, 1)$, $B(1, 1)$, $C(1, -1)$ and $D(-1, -1)$ as shown in the following diagram.



The set of vertices of the square is $V = \{A, B, C, D\}$. A reflection of the square in the line $x = 0$ can be represented by the permutation (AB)(CD) which gives the final position of the vertices.

(a) Describe the single geometric transformation represented by the permutation

(i) (ABCD);

(ii) (AC). [4]

(b) Find the permutation that represents

(i) a rotation of $180°$ about $(0, 0)$;

(ii) a reflection in the line $y = 0$. [2]

The permutations representing the eight symmetries of the square form a group $\{G, \circ\}$ where $\circ$ is the operation of composition of permutations.

(c) Find the single permutation that represents $(ABCD) \circ (AC)$. [2]

Let the identity permutation be represented by e.

(d) Find a cyclic subgroup of G of order 4. [2]

(e) Find a non-cyclic subgroup of G of order 4. [2]